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THEORETICAL AND EXPERIMENTAL STUDIES OF THE
NATURE AND CHARACTERISTICS OF SPACE-RELATED
PLASMA RESONANCE PHENOMENA

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FOREWORD

The subjects of this research grant are theoretical and experimental studies of the nature and characteristics of space-related plasma resonance phenomena. These studies have been proceeding under the direction of Prof. F. W. Crawford since the award of the grant on 1 May 1965. The current funding period is for twelve months, from 1 July 1969 to 30 June 1970, and marks the termination of the work under a separate grant. In future, some of the projects will be continued and reported on under NASA Grant NGL 05-020-176, which also has Prof. Crawford as its Principal Investigator. This is consequently both a final report and the tenth semiannual progress report on the work, covering the six-month period from 1 January to 30 June 1970.

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I. INTRODUCTION

During the five years that NASA Grant NGR 05-020-077 has been effective, the research program has evolved through three distinct phases. The general intent of the first was to elucidate the mechanisms of plasma RF pulse response phenomena observed by space-probing vehicles such as rockets and satellites. Interest in this area had been stimulated by results obtained from 1963 onwards by the Canadian topside sounder satellite, "Alouette I."^{51-57*} These indicated that shock excitation of the ionosphere by a pulsed variable frequency signal, from a transmitter carried by the satellite, was followed by prolonged ringing whenever the frequency coincided with a harmonic of the local electron cyclotron frequency, or with the local upper hybrid frequency.

Although the origin of these resonances was puzzling at first, it was soon established^{58,59} that they could be well understood in the light of warm magnetoplasma theory, and in particular in terms of the electrostatic modes known as cyclotron harmonic waves (CHW).^{60,61} The aims of the first phase of our research program were, then, to study the dispersion characteristics of these waves, both theoretically and experimentally in laboratory plasmas, and to reproduce the Alouette ringing phenomenon. This was soon done successfully for propagation perpendicular to the magnetic field.

Since the agreement between theory and experiment was very good, the possibility arose of using CHW as the basis of plasma diagnostic techniques. The second phase of our program was to examine several promising methods, including pulse transmission, resonance rectification, and impedance measurement. It is hoped that this work will point the way to future space plasma experiments involving Alouette resonances, and that the diagnostic techniques developed may be of use in the measurement of such quantities as electron density and temperature, and local magnetic field strength.

* References 1-42 are given in Section III. References 50 onwards are to be found on p 37.

Plasma wave propagation and resonance phenomena of the types just mentioned do not necessarily require electrode structures to excite them: If the plasma electron velocity distribution is non-Maxwellian, e.g., due to the presence of a group of fast electrons interacting with a background Maxwellian plasma, CHW amplification and noise emission may result. As an extension to the first phase of our research program we have been examining such possibilities recently, both experimentally and theoretically, with a view to determining whether they might occur in space plasmas where energetic electrons are known to be present. Two examples of such situations are the auroral zones and the Earth's bow shock.

As a complement to the work on plasma pulse response phenomena such as might be observed by space-probing vehicles, some attention has been given under this grant to ionospheric pulse responses observed by ground-based transmitter/receiver systems. This third phase of our program has dealt with two phenomena. The first concerns the existence and mechanism of very long delayed radio echoes, an effect first observed in the late 1920's and early 1930's²⁷ which manifested itself as reception of Morse signals with delays of up to tens of seconds after their transmission. The second concerns resonant nonlinear interaction of two signals at frequencies ω_1, ω_2 to give a radiated signal at frequency $\omega_1 \pm \omega_2$. Experimental observation of such interactions has been reported for meteor trails in auroral zones⁶² and from plasma columns in the laboratory.⁶³

The research program since inception of the grant is summarized briefly in Section II, with a rather more detailed account of the work carried out during the last six months. A bibliography of the reports, conference papers, and publications resulting from the grant is given in Section III.

II. RESEARCH PROGRAM

A. Cyclotron Harmonic Waves

Theory: Although the basic theory describing CHW had been published by Bernstein in 1958,⁶¹ no comprehensive numerical studies of their dispersion characteristics were available at the time this grant began. It was understood qualitatively that the warm magnetoplasma wave theory predicted propagation 'windows' for both electrostatic (CHW) and electromagnetic waves in the vicinity of the cyclotron harmonic frequencies, and it was believed that a wide variety of data obtained from laboratory plasma experiments on ion and electron cyclotron harmonic radiation, absorption, amplification and propagation could be interpreted in terms of this theory. The position in 1964-5 is reviewed in Ref. 60. Our work consequently began with detailed numerical studies of the CHW dispersion relation for a wide range of plasma parameters (ω_p , ω_c), and electron velocity distributions.^{9,20,25} These ranged from stable Maxwellian plasmas to delta-function beams interacting with Maxwellian plasmas, and covered propagation both perpendicular and oblique to the static magnetic field. The overall results are best summarized in Refs. 31 and 32.

It should be emphasized that computation of CHW dispersion relations is by no means trivial, particularly when collisional and/or collisionless (cyclotron) damping is appreciable.^{1,17} Contributions were made in this early work to improving the formalism, both for obtaining dispersion relations rapidly and in expressing them in forms suitable for economical computation.^{11,12,18} Our interests were always directed towards conditions which might be realized experimentally, and it was concluded that although many velocity distributions might occur experimentally only two were of major importance for readily controllable laboratory conditions. These were the Maxwellian, and the combination of an electron beam interacting with a Maxwellian background plasma. The first is relevant to the "Alouette" resonances observed in the ionosphere. The second gives rise to a very basic example of CHW instability. Both were given special attention in relation to our laboratory program.

Experiment: A feature peculiar to CHW in a Maxwellian plasma is that propagation perpendicular to the static magnetic field is not subject to collisionless damping. It should consequently be possible to carry out Alouette-type 'ringing' experiments with a single antenna oriented parallel to the magnetic field, shock-excited at frequencies for which the CHW group velocity is predicted to be zero. Alternatively group delay measurements can be made for propagation across the magnetic field to a second antenna. Both types of experiments were carried out successfully.^{5,8,10,13,14,21,22} They succeeded in verifying CHW theory for the first time, and in so doing contributed valuable support to microscopic plasma wave theory in general.²⁸ The ringing observations confirmed the suggested explanation of "Alouette" resonances as a zero group velocity effect. The only significant point left outstanding was that of the temporal decay of the ringing which did not check closely with theory based on rather idealized models.^{26,51}

The theory for an electron beam interacting with a plasma was examined in detail numerically for various hypothetical combinations of hot or cold background plasma with hot or cold beams, i.e. with beams having varying transverse and longitudinal energy spread.³³ It was found that CHW growth was generally predicted for propagation almost perpendicular to the static magnetic field, and that this instability was expected to occur in competition with longitudinal beam-plasma interaction. Some experiments were carried out under another contract to demonstrate that the theory predicted noise amplification in frequency ranges corresponding to those actually observed.⁶⁴ Close agreement was impossible to obtain since the growth was absolute rather than convective, i.e. signals grew temporally from noise to saturation, rather than obeying the simple linear theory for an externally-applied signal. There was in addition the competing influence of longitudinal beam-plasma interaction mentioned above. As an outgrowth of these beam-plasma experiments, attention was focused on this longitudinal beam-plasma mechanism under the present grant. As these experiments do not strictly concern CHW, discussion will be deferred to Section IID.

B. Diagnostic Techniques

In view of the success of our group delay measurements on CHW wave-packets, it seemed likely that a technique might be feasible for determining plasma electron density, temperature, and cyclotron frequency, free from local effects such as plasma sheaths, simply by fitting CHW dispersion curves to group delay data for two antennas a known distance apart perpendicular to the magnetic field. A suitable experiment was proposed for a NASA orbiting workshop project, but after further examination and model experiments it seems likely that results would be difficult to obtain in practice due to mechanical and alignment problems with the necessary antennas. A more attractive alternative is to measure the plasma impedance between two electrodes as a function of frequency, and to deduce the plasma parameters from such measurements. This is effectively a magnetoplasma capacitor experiment, and considerable attention has been given under the grant to the boundary-value problem posed by an electrode system filled with a hot magnetoplasma.

Our first approach to the plasma capacitor problem was to simply assume a planar or coaxial system filled with plasma having the permittivity predicted by Bernstein. This in turn predicted a series of resonances damped only by collisions.^{3,4} However, experiment showed that these did not occur unless grids were used for the electrodes. The reason for this can be traced to collisionless damping associated with open charged particle orbits when opaque electrodes are used. For this reason, a full orbit analysis was carried out which demonstrated increased damping and disappearance of the resonances.^{24,39} This work has not been submitted for publication yet, although most of it was completed over two years ago. Recently, we have been working on a revised version of the original report. It should be emphasized in connection with this work that it is for planar and coaxial geometry, neither of which are attractive for laboratory or space plasma experiments. More convenient would be parallel wire geometry, and we have begun to extend the theory during the last reporting period to cover this case.

Some years ago, a useful diagnostic technique was developed based on resonance rectification.⁶⁵ This was primarily for unmagnetized plasmas. A spherical probe surrounded by its space-charge sheath could be shown to resonate with the plasma at a frequency related to the local electron plasma frequency. Coinciding with the RF resonance is a peak in the nonlinearly rectified DC probe current. A similar effect is to be expected for a wire probe aligned parallel to the static magnetic field in a hot magnetoplasma. Work carried out under the grant demonstrated a series of such resonances which could be used to diagnose the properties of the plasma.^{19,23}

C. Ionospheric Pulse Response

The "Alouette" resonances stimulating the first phase of our program were excited by space-probing vehicles. In contrast, amongst our more recent projects have been two related to ionospheric response to pulses transmitted from ground-based transmitters, observed with ground-based receivers. The first concerns very long delayed radio echoes (LDE), and the other nonlinear mixing in meteor trails.

LDE: During the 1969 contract year, some support was derived from this grant for ionospheric probing studies aimed at observing LDE. This phenomenon was first observed in the late 1920's and early 1930's, and manifested itself as reception of Morse signals with delays of up to tens of seconds after their transmission.²⁷ Motivation to re-examine the effect, and to establish its origin, was provided by observation of 'memory' phenomena in laboratory plasmas at Stanford and elsewhere. It seemed that the mechanism operative in these experiments might account for LDE. Closer examination showed this not to be so, and two alternative mechanisms were studied: small-signal growth of ordinary waves, and parametric amplification. In parallel with the theoretical results, an ionospheric sounding system was run under the present grant with the object of obtaining evidence for the existence of LDE. The results were initially negative, and the work was transferred to an NSF grant obtained specifically to cover the project. Since then, about a dozen records have been obtained which appear to show LDE, and a project related to them

will probably be included in the extension program to the present grant to be carried out under NASA Grant NGL 05-020-176.

Meteor Trails: There is currently a very rapid growth of interest in nonlinear wave phenomena. This stems from two factors: first, the successful verification of nearly all basic small-signal wave dispersion characteristics, either in space or in laboratory plasmas,²⁸ and second, an accumulation of data from high-power experiments which seem to be explicable only in terms of nonlinear theory. It is clear that projects such as the ESSA 1 MW sounder at Boulder, Colorado, will rapidly provide a series of nonlinear problems for solution. We have chosen for study under this grant the nonlinear scattering from meteor trails. This seems to have been observed already in the ionosphere,⁶² and lends itself to analysis and to ready simulation in a laboratory experiment as a nonlinear scattering experiment on a positive column. Some initial theoretical work was described in SAR 9.⁴¹ Progress during the last six months of the grant is as follows.

Theory: We have extended the theory for a homogeneous plasma column described in SAR 9 to include radial inhomogeneity. The work will be included in a comprehensive report now in preparation³⁸ so only an outline will be given here. The situation considered is that of an infinitely long column, oriented along the z-axis, in which the plasma density varies only with radius. Plane waves with frequencies ω_β and ω_γ propagating along the x-axis in the direction of increasing x interact nonlinearly in the column to produce a third wave at frequency $\omega_\alpha (= \omega_\beta + \omega_\gamma)$. The impinging waves have E_y and H_z components only, while the scattered wave has only H_z , E_θ , and E_r components. All field quantities are assumed to depend only on x and y (or r and θ). Cold plasma theory, and Maxwell's equations with the quasistatic approximation, form the framework of the theory.

A plane wave travelling as described sets up a potential field of the form

$$V_{inc} = \bar{E} r \sin \theta, \quad (1)$$

where θ is the angle between r and the x axis. Outside the plasma column, of radius a , the voltage satisfies Laplace's equation ($\nabla^2 V = 0$), which has a solution of the form

$$V = \bar{E} \left(a \xi + \frac{A}{\xi} \right) \sin \theta , \quad (2)$$

where $\xi = r/a$. Inside, the governing equation is

$$K \nabla^2 V + \nabla K \cdot \nabla V = 0 , \quad (3)$$

where $K = 1 - \omega_p^2 / \omega^2$, and ω_p is the electron plasma frequency. The linear solution to Eq. (3) may be written as

$$V = g \psi \bar{E} a \sin \theta , \quad (4)$$

where g satisfies the equation

$$\frac{d^2 g}{d\xi^2} + \frac{1}{\xi} \frac{dg}{d\xi} - \frac{g}{\xi^2} + \frac{1}{K} \frac{dK}{d\xi} \frac{dg}{d\xi} = 0 \quad (5)$$

with $g = 0$, $g' = 1$, and $g'' = 0$ at $\xi = 0$, and the prime refers to differentiation with respect to ξ . When we apply the boundary conditions that V and $K \partial V / \partial \xi$ are continuous we find that

$$\psi = \frac{2}{(g + K g')_i} , \quad (6)$$

where subscript i refers to the inside of the boundary at $r = a$.

Inside the plasma, the electric field is given by

$$E_r = \bar{E} \psi g' \sin \theta , \quad E_\theta = \bar{E} \psi g \cos \theta / \xi . \quad (7)$$

The velocity \tilde{v} is given by

$$\tilde{v} = \frac{\eta}{j\omega} \tilde{E} , \quad (8)$$

where η is the electron charge-to-mass ratio.

The charge density is obtained by solving the equation

$$\mathbf{K} \cdot \nabla \cdot \underline{\underline{\mathbf{E}}} + \nabla \cdot \mathbf{K} \cdot \underline{\underline{\mathbf{E}}} = 0 \quad (9)$$

for $\nabla \cdot \underline{\underline{\mathbf{E}}}$ to obtain

$$\rho = \epsilon_0 \nabla \cdot \underline{\underline{\mathbf{E}}} = - \epsilon_0 \frac{\nabla \mathbf{K} \cdot \underline{\underline{\mathbf{E}}}}{\mathbf{K}} = \frac{\epsilon_0 \bar{\mathbf{E}} \psi}{a} s \sin \theta , \quad (10)$$

where s has been written for $-g'K'/K$. The surface charge is found from the relation

$$\rho_s = [\epsilon_0 (K-1) \bar{\mathbf{E}}]_i = [\epsilon_0 \bar{\mathbf{E}} g' \psi (K-1) \sin \theta]_i . \quad (11)$$

We now turn to the nonlinear theory. Our basic equations are Maxwell's equations

$$\nabla \times \underline{\underline{\mathbf{H}}} = \epsilon_0 \frac{\partial \underline{\underline{\mathbf{E}}}}{\partial t} + \underline{\underline{\mathbf{J}}} , \quad \nabla \times \underline{\underline{\mathbf{E}}} = 0 , \quad \nabla \cdot \underline{\underline{\mathbf{E}}} = \rho / \epsilon_0 , \quad (12)$$

together with the momentum transfer equation, and the current equation

$$\frac{\partial \underline{\underline{\mathbf{v}}}}{\partial t} + (\underline{\underline{\mathbf{v}}} \cdot \nabla) \underline{\underline{\mathbf{v}}} = \eta \underline{\underline{\mathbf{E}}} , \quad \underline{\underline{\mathbf{J}}} = \rho \underline{\underline{\mathbf{v}}} . \quad (13)$$

If we expand these equations in a Fourier series with terms of the form $\exp j(\omega_\alpha t - \mathbf{k}_\alpha \cdot \mathbf{x})$, they take the form

$$\nabla \times \underline{\underline{\mathbf{H}}}_\alpha = j\omega_\alpha \epsilon_0 \underline{\underline{\mathbf{E}}}_\alpha + \underline{\underline{\mathbf{J}}}_\alpha , \quad \nabla \times \underline{\underline{\mathbf{E}}}_\alpha = 0 , \quad \nabla \cdot \underline{\underline{\mathbf{E}}}_\alpha = \rho_\alpha / \epsilon_0 , \quad (14)$$

$$j\omega_\alpha \underline{\underline{\mathbf{v}}}_\alpha = \eta \underline{\underline{\mathbf{E}}}_\alpha - \Sigma (\underline{\underline{\mathbf{v}}}_\beta \cdot \nabla) \underline{\underline{\mathbf{v}}}_\gamma , \quad \underline{\underline{\mathbf{J}}}_\alpha = \rho_0 \underline{\underline{\mathbf{v}}}_\alpha + \Sigma \rho_\beta \underline{\underline{\mathbf{v}}}_\gamma ,$$

where $\omega_\alpha = \omega_\beta + \omega_\gamma$, and Σ is the symmetry operator defined by the equation

$$\Sigma f(\beta, \gamma) = f(\beta, \gamma) + f(\gamma, \beta) .$$

If we substitute the last two expressions in Eq. (14) into the first, and simplify, we obtain

$$\nabla \times \underline{\underline{\mathbf{H}}}_\alpha = j\omega_\alpha \epsilon_0 \mathbf{K}_\alpha \underline{\underline{\mathbf{E}}}_\alpha + \Sigma \rho_\beta \underline{\underline{\mathbf{v}}}_\gamma + \frac{j\rho_0}{\omega_\alpha} \nabla (\underline{\underline{\mathbf{v}}}_\beta \cdot \underline{\underline{\mathbf{v}}}_\gamma) . \quad (15)$$

If we divide by K_α , and take the curl of this equation, we obtain

$$\nabla_{\alpha}^2 \tilde{H}_\alpha - K_\alpha \nabla \frac{1}{K_\alpha} \times (\nabla \times \tilde{H}_\alpha) = - K_\alpha \left\{ \Sigma \nabla \left(\frac{\rho_\beta}{K_\alpha} \right) \times \tilde{v}_\gamma + \nabla \left(\frac{j\rho_0}{\omega K_\alpha} \right) \times \nabla (\tilde{v}_\beta \cdot \tilde{v}_\gamma) \right\}. \quad (16)$$

In obtaining Eq. (16), we have used the fact that in nonlinear terms we must substitute first order quantities, and that the first order \tilde{v} and \tilde{E} satisfy the relations $\tilde{v} = \eta \tilde{E} / j\omega$ and $\nabla \times \tilde{v} = 0 = \nabla \times \tilde{E}$. We can write out Eq. (16) as

$$\sin 2\theta \left[\frac{\partial^2 H_{z\alpha}}{\partial \xi^2} + \frac{1}{\xi} \frac{\partial H_{z\alpha}}{\partial \xi} - \frac{H_{z\alpha}}{\xi^2} - \frac{1}{K_\alpha} \frac{dK_\alpha}{d\xi} \frac{\partial H_{z\alpha}}{\partial \xi} \right] = T, \quad (17)$$

$$\hat{z}T = - a^2 K_\alpha \left\{ \Sigma \nabla \left(\frac{\rho_\beta}{K_\alpha} \right) \times \tilde{v}_\gamma + \nabla \frac{j\rho_0}{\omega K_\alpha} \times \nabla (\tilde{v}_\beta \cdot \tilde{v}_\gamma) \right\}.$$

We are now in a position to derive the expression for T . Using Eqs. (7), (8), and (10) we see that

$$\nabla \left(\frac{\rho_\beta}{K_\alpha} \right) = \frac{\epsilon_0}{a^2} \left[\left(\frac{s'_\beta}{K_\alpha} - \frac{s_\beta K'_\alpha}{K_\alpha^2} \right) \sin \theta \hat{r} + \frac{s_\beta \cos \theta}{\xi K_\alpha} \hat{\theta} \right] \bar{E}_\beta \psi_\beta,$$

$$\nabla \left(\frac{\rho_\beta}{K_\alpha} \right) \times \tilde{v}_\gamma = \frac{\eta \epsilon_0 \sin 2\theta}{2j\omega_\gamma a^2 \xi K_\alpha} \left\{ s'_\beta g_\gamma - s_\beta g'_\gamma - s_\beta g_\gamma \frac{K'_\alpha}{K_\alpha} \right\} \bar{E}_\beta \bar{E}_\gamma \psi_\beta \psi_\gamma \hat{z}. \quad (18)$$

Similarly, we obtain

$$\nabla (\tilde{v}_\beta \cdot \tilde{v}_\gamma) = - \frac{\eta^2}{\omega_\beta \omega_\gamma} \nabla \left(g'_\beta g'_\gamma \sin^2 \theta + g_\beta g_\gamma \frac{\cos^2 \theta}{\xi^2} \right) \psi_\beta \psi_\gamma \bar{E}_\beta \bar{E}_\gamma, \quad (19)$$

or, alternatively,

$$\left. \nabla(\vec{v}_\beta \cdot \vec{v}_\gamma) \right|_\theta = -\frac{\eta^2 \sin 2\theta}{\omega_\beta \omega_\gamma a \xi} \left(\vec{g}'_\beta \vec{g}'_\gamma - \frac{\vec{g}_\beta \vec{g}_\gamma}{\xi^2} \right) \psi_\beta \psi_\gamma \bar{\vec{E}}_\beta \bar{\vec{E}}_\gamma . \quad (20)$$

Substituting into Eq. (17) yields

$$T = \frac{\eta j \epsilon_0}{\omega_N} \psi_\beta \psi_\gamma \bar{\vec{E}}_\beta \bar{\vec{E}}_\gamma \sin 2\theta R , \quad (21)$$

$$R = \omega_N \left\{ \Sigma \left(\frac{\vec{s}'_\beta \vec{g}_\gamma - \vec{s}_\beta \vec{g}'_\gamma - \vec{s}_\beta \vec{g}_\gamma \frac{K'_\alpha}{K_\alpha}}{2\omega_\gamma \xi} \right) - \frac{K_\alpha}{\omega_\alpha \omega_\beta \omega_\gamma} \frac{d}{d\xi} \left(\frac{\omega_p^2}{K_\alpha} \right) \left(\frac{\vec{g}_\beta \vec{g}_\gamma}{\xi^2} - \frac{\vec{g}'_\beta \vec{g}'_\gamma}{\xi} \right) \right\} ,$$

where ω_N is some arbitrary reference frequency.

The first boundary condition is given by

$$H_{zi} - H_{z0} = J_s , \quad (22)$$

where subscript i and 0 refer to the inner and outer sides of the boundary at $\xi = 1$, and J_s is the surface current. The surface current is given by

$$J_s = \Sigma \rho_{s\beta} v_{\theta\gamma} , \quad (23)$$

which, upon substitution of Eqs. (7), (8), and (11), takes the form

$$J_s = \Sigma \bar{\vec{E}}_\beta \bar{\vec{E}}_\gamma \psi_\beta \psi_\gamma \frac{\eta \epsilon_0}{2j\omega_\gamma} \sin 2\theta \left[(K_\beta - 1) \vec{g}'_\beta \vec{g}_\gamma \right]_i . \quad (24)$$

The second boundary condition comes from the equality of \vec{E}_θ on either side of the boundary. Using Eq. (15), this states that

$$K_{\alpha i} \frac{\partial H_{z\alpha}}{\partial r} \Big|_0 = \left[\frac{\partial H_{z\alpha}}{\partial r} + \Sigma \rho_{\beta} v_{\theta\gamma} + \frac{j\rho_0}{\omega_\alpha} \left[\nabla(\vec{v}_\beta \cdot \vec{v}_\gamma) \right]_\theta \right]_i . \quad (25)$$

Substituting Eqs. (7), (8), and (10), we obtain

$$K_{\alpha i} \left. \frac{\partial H_{z\alpha}}{\partial \xi} \right|_0 = \left. \frac{\partial H_{z\alpha}}{\partial \xi} \right|_i + \frac{\eta \epsilon_0 j}{\omega_N} \psi_\beta \psi_\gamma \bar{E}_\beta \bar{E}_\gamma \sin 2\theta Q, \quad (26)$$

$$Q = - \omega_N \left\{ \Sigma \frac{s_\beta g_\gamma}{2\omega_\gamma} + \frac{\omega^2}{\omega_\alpha \omega_\beta \omega_\gamma} (g'_\beta g'_\gamma - g_\beta g_\gamma) \right\}_i.$$

We solve for $H_{z\alpha}$ by noting that outside the plasma the field has the form

$$H_{z\alpha} = \bar{E}_\beta \bar{E}_\gamma B \sin 2\theta H_2^{(2)} \left(\frac{\omega_\alpha r}{c} \right) \simeq \frac{4jB}{\pi} \sin 2\theta \left(\frac{c}{\omega_\alpha r} \right)^2 \bar{E}_\beta \bar{E}_\gamma, \quad (27)$$

while its derivative is given by

$$\frac{dH_{z\alpha}}{d\xi} \simeq \frac{8jB}{\pi} \sin 2\theta \left(\frac{c^2 a}{\omega_\alpha^2 r^3} \right) \bar{E}_\beta \bar{E}_\gamma, \quad (28)$$

where B is a constant to be determined. From Eqs. (17) and (21), we see that the field inside the plasma is given by

$$H_{z\alpha} = \bar{E}_\beta \bar{E}_\gamma \left[Ah_{1\alpha} + \frac{\eta j \epsilon_0}{\omega_N} \psi_\beta \psi_\gamma h_{2\alpha} \right] \sin 2\theta, \quad (29)$$

where A is a constant; $h_{1\alpha}$ satisfies the equation

$$\left(\frac{d^2}{d\xi^2} + \frac{1}{\xi} \frac{d}{d\xi} - \frac{4}{\xi^2} - \frac{K'_\alpha}{K_\alpha} \frac{d}{d\xi} \right) h_{1\alpha} = 0, \quad (30)$$

with $h_{1\alpha} = 0$, $h'_{1\alpha} = 0$, and $h''_{1\alpha} = 1$ at $\xi = 0$, and $h_{2\alpha}$ satisfies the equation

$$\left(\frac{d^2}{d\xi^2} + \frac{1}{\xi} \frac{d}{d\xi} - \frac{4}{\xi^2} - \frac{K'_\alpha}{K_\alpha} \frac{d}{d\xi} \right) h_{2\alpha} = R , \quad (31)$$

with $h_{2\alpha} = h'_{2\alpha} = h''_{2\alpha} = 0$ at $\xi = 0$. If we substitute Eqs. (27), (28), and (29) into the boundary conditions, we can show that B is given by

$$B = \frac{\eta \epsilon_0 \psi_\beta \psi_\gamma \pi \omega_\alpha^2 a^2}{4\omega_N c^2} \sigma , \quad (32)$$

$$\sigma = \left[\frac{h'_{1\alpha} h_{2\alpha} - h_{1\alpha} (h'_{2\alpha} + Q) + \Sigma \frac{(K_\beta - 1) g'_\beta g_\gamma h'_{1\alpha}}{2\omega_\gamma / \omega_N}}{h'_{1\alpha} + 2K_\alpha h_{1\alpha}} \right]_i .$$

In order to relate B to the incoming and outgoing power fluxes, we note that the incoming power is given by

$$P_{inc} = 2 \epsilon_0 c \overline{E}^2 , \quad (33)$$

while the outgoing power associated with the wave corresponding to Eq. (27) is given by

$$P_{out} = \frac{4B^2 \sin^2 2\theta}{\pi} \left(\frac{\mu_0}{\epsilon_0} \right)^{1/2} \left(\frac{c}{\omega_\alpha a} \right) \overline{E}_\beta^2 \overline{E}_\gamma^2 . \quad (34)$$

Substituting from Eqs. (32) and (33), we obtain the final result

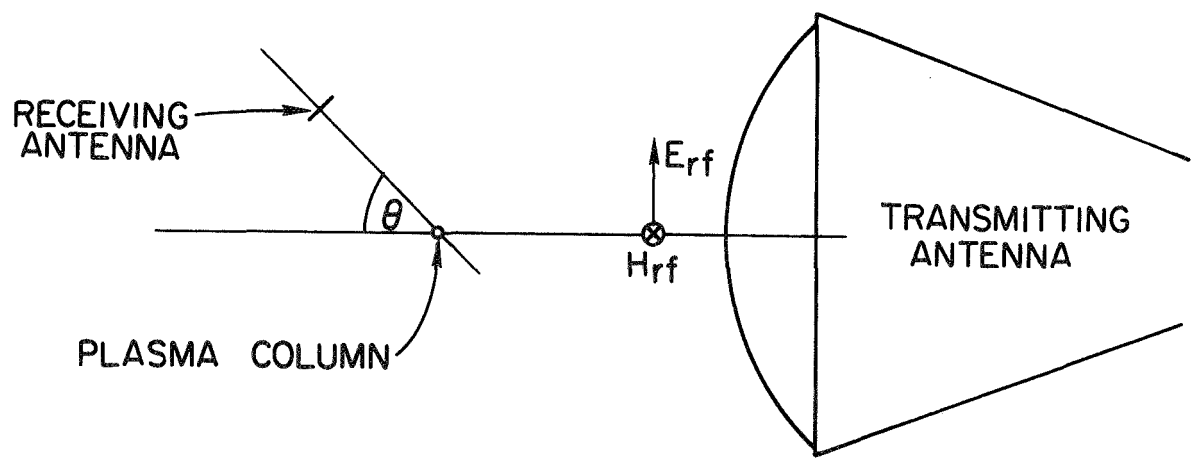
$$P_{out} = \frac{\chi}{4\omega_N^2} \left(\frac{\omega_N a}{c} \right)^3 \sin^2 2\theta P_\beta P_\gamma , \quad \chi = \frac{\eta^2 \pi}{4c^3 \epsilon_0} \left(\frac{\omega_\alpha}{\omega_N} \right)^3 (\psi_\beta \psi_\gamma \sigma)^2 . \quad (35)$$

Recently, we have begun to carry out computations of Eq. (35) to compare the scattering from uniform and parabolic plasma profiles. This work will be reported in the next SAR under grant NGL 05-020-176.

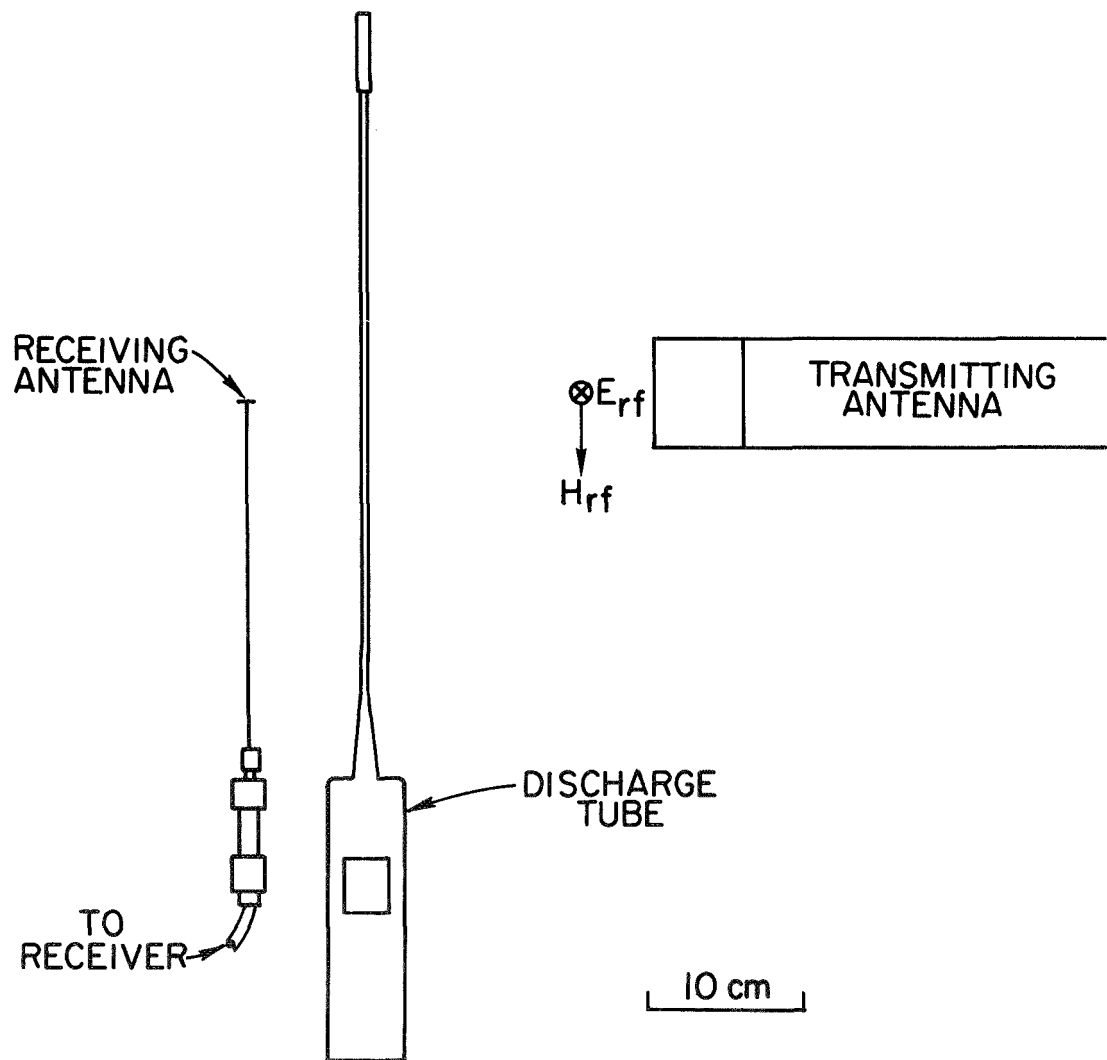
Experiment: Estimates of scattered power suggest that the theory just outlined should be readily susceptible to laboratory verification. If this can be carried out successfully, it should be possible to decide whether the effect could explain the ionospheric observations of frequency mixing in meteor trails.⁶² Two experimental approaches suggest themselves: first, to suspend a plasma column in free space, and to illuminate with signals at $\omega_\beta, \omega_\gamma$. A movable horn can then be used to measure the scattered power at ω_α , and the polar pattern. Second, is the possibility of placing the tube in a microwave cavity resonant at $\omega_\alpha, \omega_\beta$, and ω_γ , and measuring the power output at ω_α . We chose the second for our initial studies, since the use of a cavity allows high electric fields to be applied to the plasma. For simplicity, a cavity resonant in the TE_{111} and TE_{121} modes for $\omega_\alpha/2 = \omega_\beta = \omega_\gamma$ was designed. It was found to be extremely difficult to adjust for optimum performance, so this method was abandoned in favor of scattering from a column suspended effectively in free space.

Figure 1 shows the arrangement of the plasma and antennas used in our experiments. The plasma is a mercury-vapor positive column in a small pyrex tube (outside diameter 5 mm, inside diameter 3 mm). The transmitting antenna is an S-band horn with a dielectric lens. The receiving antenna is a 24 mm dipole supported on small diameter 50 Ω rigid coax line.

Figure 2 shows the signal generating and receiving system. In this, the transmitter signal is amplitude-modulated by a 1 kHz square wave. The 1 kHz signal at the receiver is amplified and detected in a narrow band lock-in amplifier. This permits the use of a much smaller passband than is provided by the receiver IF amplifier. It would not be possible to use a very narrow band in the IF because the microwave oscillators (2.20 GHz and 4.43 GHz) are frequency-modulated by power supply ripple, and have bandwidths of several hundred kHz. The advantage of a narrow band system is less noise, and hence greater sensitivity. In this



(a) PLAN VIEW



(b) SIDE VIEW

FIG. 1. SET-UP FOR STUDYING SCATTERING FROM A PLASMA COLUMN.

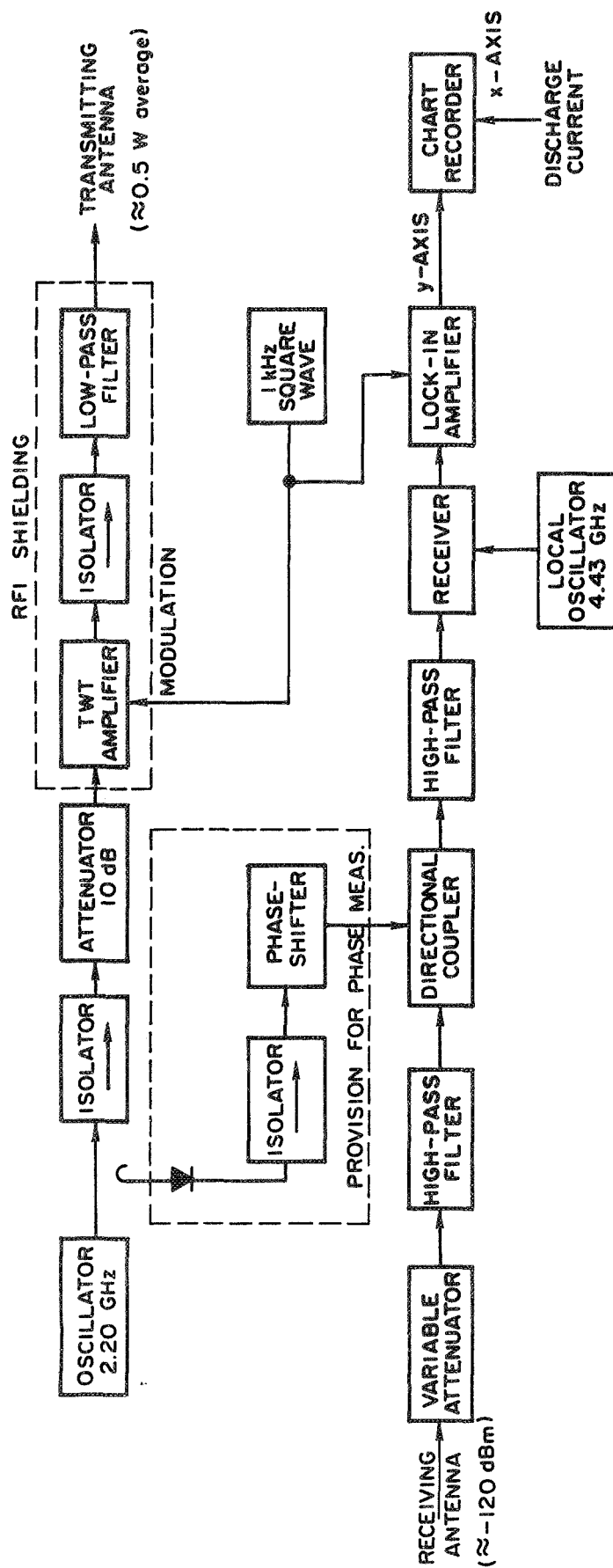


FIG. 2. SCHEMATIC OF TRANSMITTING AND RECEIVING SYSTEM FOR SCATTERING STUDIES.

experiment, high sensitivity is needed. A further improvement in signal-to-noise ratio can be obtained by injecting a small amount of 4.40 GHz CW into the receiver. The system then becomes sensitive to the phase of the received second harmonic and the output is one of the two quadrature components of the input. The other component is obtained by changing the phase shifter by 90° . In the experimental work thus far, this provision for phase measurements has not been used extensively.

With the system just described, the power of the RF field incident on the plasma (during the "on" half of the modulating square wave) is 2 mW/cm^2 . Its value was measured by comparison with waveguide fields of known power. Figure 3 shows typical data plots for this configuration. In the plots, the second harmonic amplitude is recorded as a function of discharge current. The amplitude characteristic of the system is very nearly square-law so that the Y-axis on the plots represents signal power. At a current of 130 mA, the plasma column is dipole resonant at the applied frequency (2.20 GHz). At this point there is maximum harmonic generation. Figure 3(a) shows this resonance. Harmonic generation has also been observed at higher order resonances (Tonks-Dattner resonances) as shown in Fig. 3(b).

In the course of our preliminary experiments, it was noted that weak magnetic fields, the Earth's magnetic field in particular, had a strong effect on the second harmonic radiation pattern. This effect does not seem to have been mentioned before in the literature. The presence of a magnetic field produced a strong component of dipolar radiation. This suggests that agreement with theory can only be expected after the Earth's field has been cancelled. Recently, we have been taking scattering data with the Earth's magnetic field cancelled by applying an equal but opposite field with a large diameter (≈ 1 meter) 25-turn coil. Without such a coil, the radiation pattern is essentially dipolar. With the coil, quadrupole radiation is observed, as suggested by theory. Detailed polar patterns are being taken and will be presented in the next SAR under NASA Grant NGL 05-020-176.

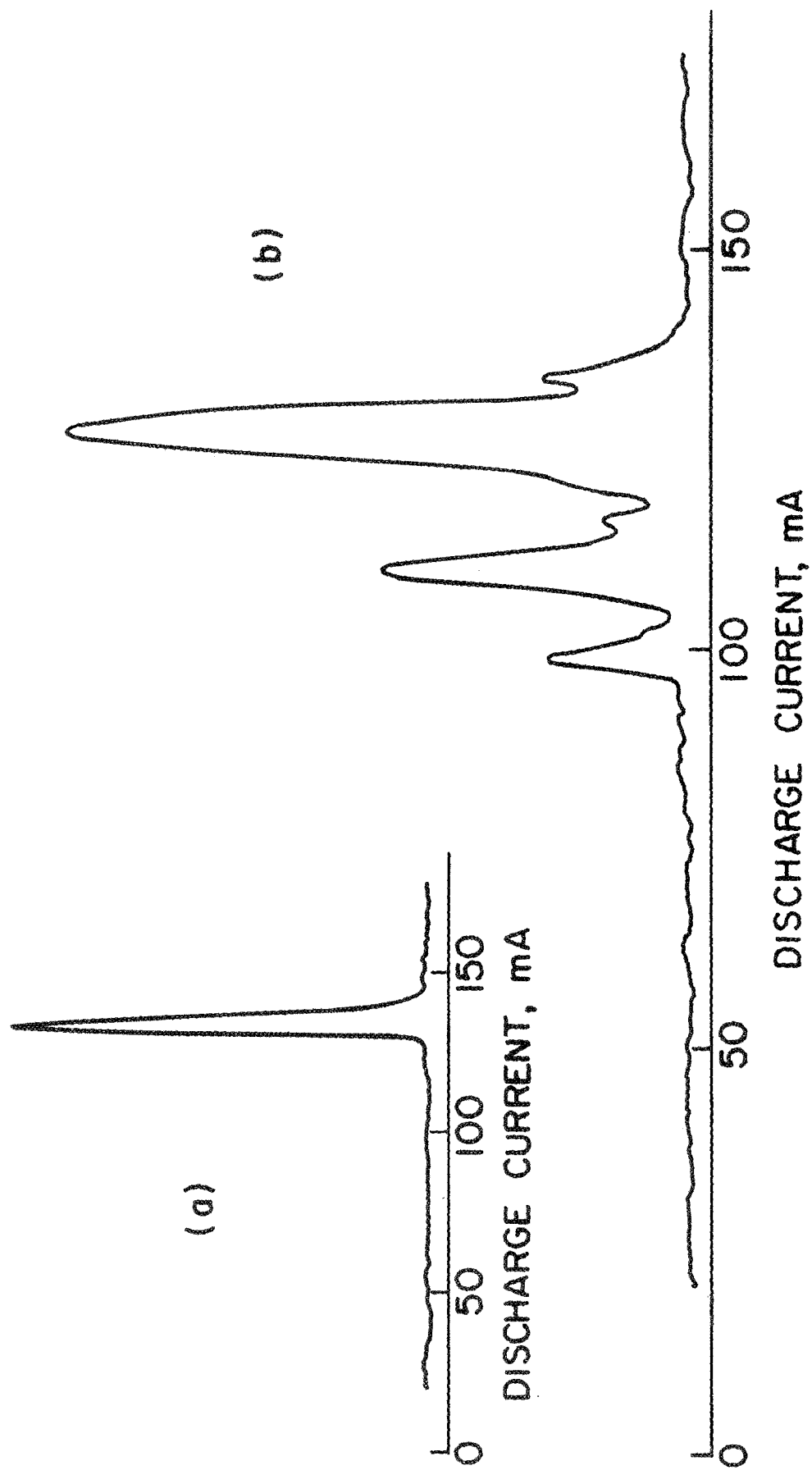


FIG. 3. TYPICAL EXPERIMENTAL SCATTERING DATA.

D. Beam-plasma Interaction

As was mentioned in Section IIA, difficulty was encountered when trying to obtain detailed comparisons between theory and experiment for electron beam-stimulated CHW. This was due to longitudinal beam-plasma interaction leading to strong growth near the electron plasma and cyclotron frequencies (ω_p, ω_c). This type of amplification feeds on the beam energy parallel to the magnetic field, whereas CHW amplification relies on extraction of perpendicular energy, i.e., that due to gyration of the beam electrons about magnetic field lines. In important ionospheric situations, such as the auroral zones and the earth's bow shock, groups of electrons interact with the background plasma and may be responsible for both types of amplification. For this reason, effort under the grant has been applied to elucidating certain of their features.

The first project was that described in Section IIA. This was followed by some theoretical work described in SAR 9⁴¹ whose objects were to establish how longitudinal growth causes spreading of an initial monoenergetic beam velocity distribution. This work was carried out by a visiting research associate (Dr. H. J. Hopman) who had previously obtained experimental measurements of the velocity spreading for comparison with the theory.^{66,67} The significance of this work is that as the axial velocity spread increases, the growth due to longitudinal beam-plasma interaction decreases until finally the dominant observable effects are those due to stimulation of perpendicularly propagating CHW. Although the qualitative agreement is satisfactory, there are significant differences between the theory carried out for an infinite beam, and experiments for a necessarily finite beam.

A remarkable feature of beam-plasma interaction experiments carried out with magnetoplasmas is that RF noise is generally emitted in sharp irregular bursts. Temporal growth from noise is not unexpected from the theory, but it is still unexplained why the signals shut off and repeat, rather than growing to a steady state of saturation. The solution is obviously not contained by linear theory, so we have undertaken a study during the last six months of the grant of nonlinear beam-plasma interaction, with the intention of predicting the temporal nonlinear beam-

plasma interaction behavior and checking it experimentally in an apparatus designed for beam-plasma studies under another contract which has become available to us.⁶⁸ It is hoped that the results will be relevant to ionospheric phenomena such as time-varying noise emissions from the auroral zones. Progress during the final reporting period is as follows.

Theory: It was pointed out by Sturrock,⁶⁹ nearly ten years ago, that if three waves propagating as $\exp j(\omega t - \underline{k} \cdot \underline{r})$ interact nonlinearly and have frequencies and wave numbers satisfying the resonance conditions,

$$\omega_{\alpha} = \omega_{\beta} + \omega_{\gamma} , \quad \underline{k}_{\alpha} = \underline{k}_{\beta} + \underline{k}_{\gamma} \quad (36)$$

then, provided that the sign of the small-signal energy of Wave α differs from that of β and γ , explosive instability can occur, i.e. growth from noise to infinite amplitude can occur in a finite time. Naturally, higher order corrections will prevent infinite amplitude actually being reached, but there is a strong implication that experimental situations should occur in which this faster-than-exponential nonlinear growth should dominate over small-signal exponential growth. The question naturally arises of whether this is the case for beam-plasma interaction in magnetoplasmas, and can explain the noise bursts commonly observed from them. It is this question that we have set out to answer.

From the experimental point of view, it would be desirable to study nonlinear explosive instabilities under conditions which are stable according to linear theory. That such situations should be attainable is indicated by Fig. 4, which shows dispersion characteristics for monoenergetic (delta-function) electron beam and cold plasma velocity distributions,

$$f_{0j} = \delta(v_z - v_{0j}) \delta(v_r) \delta(v_{\theta}) , \quad (37)$$

under conditions where the beam and plasma fill a conducting tube of radius a and $\omega_{ce} \gg \omega_{pe}$. The corresponding dispersion relation is

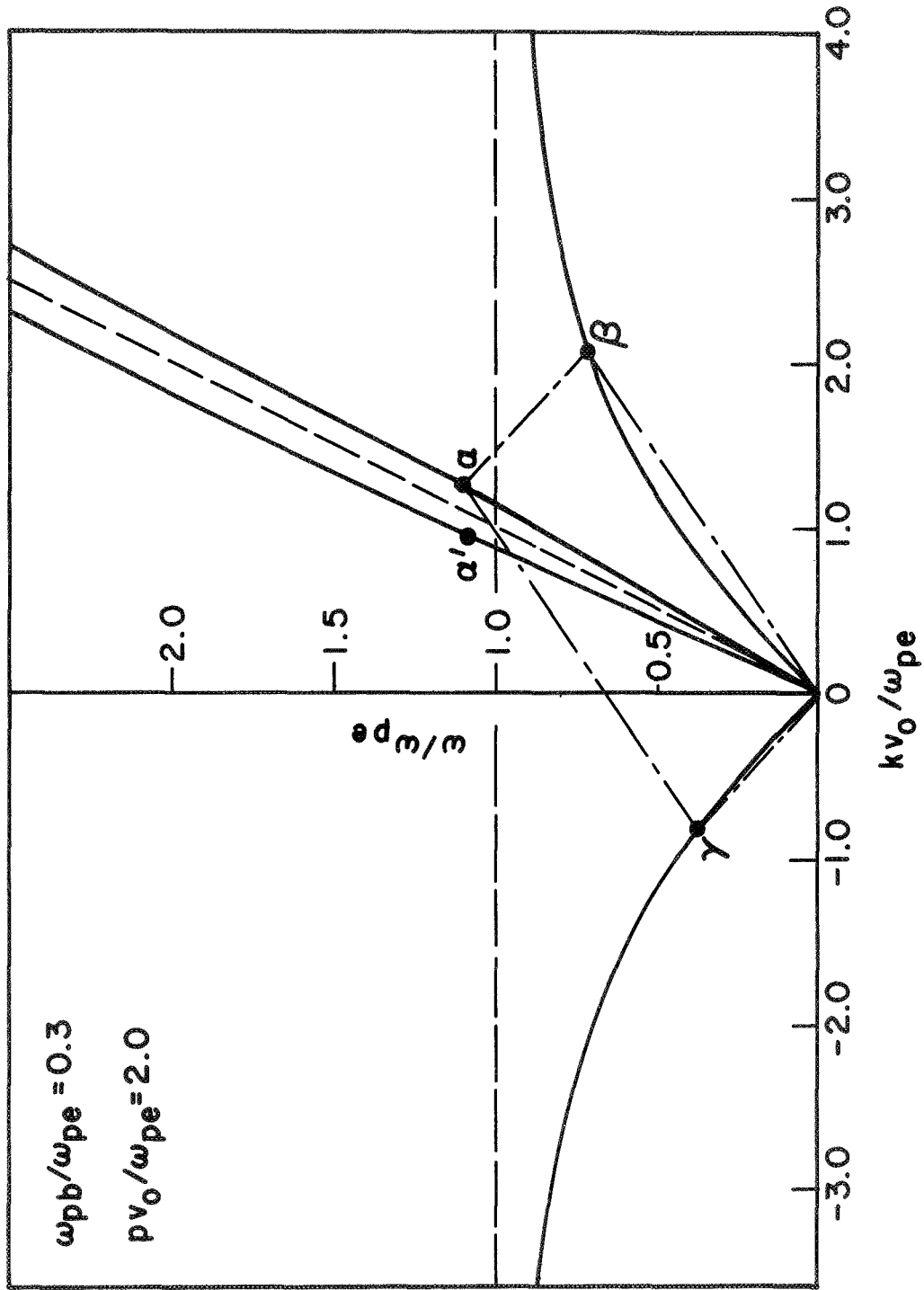


FIG. 4. DISPERSION DIAGRAM OF A STABLE BEAM-PLASMA SYSTEM. WAVES α , β , AND γ FULFILL THE SYNCHRONISM CONDITIONS FOR NONLINEAR THREE-WAVE INTERACTIONS

$$D_{\alpha} = 1 - \frac{1}{p^2 + k_{\alpha}^2} \sum_j \frac{\omega_{pj}^2}{(v_{\alpha} - v_{0j})^2}, \quad (38)$$

where p_{α} is a J_m Bessel function root. Its lowest value is 2.4 for an axisymmetric mode. For the condition

$$\frac{pv_{0b}}{\omega_{pe}} > \left[1 + \left(\frac{\omega_{pb}}{\omega_{pe}} \right)^{2/3} \right]^{3/2}, \quad (39)$$

the two beam modes do not intersect the plasma modes and the system is small-signal stable. As the parallelogram with vertices $\alpha\beta\gamma 0$ demonstrates, the synchronism conditions of Eq. (36) can be satisfied and nonlinear growth should occur. This will be explosive since the slow beam wave carries negative RF energy while the plasma waves carry positive energy. On the other hand, a parallelogram with α' as a vertex, lying on the fast beam wave branch, involves three positive energy modes. For these conditions, parametric amplification should be observable, but the growth will not be explosive.

During the reporting period we have carried out an analysis to determine the parametric growth rate. Details will be given in a report now in preparation.⁴⁰ Here, we will simply quote the salient results. We find that there will be temporal growth of the β and γ waves as $\exp \Omega t$, when Wave α is the pump, with

$$\Omega^2 = \frac{4A^2 |\Gamma_{\alpha\beta\gamma}|^2 |\hat{v}_{\alpha}|^2}{\left(p^2 + k_{\beta}^2 \right) \left(p^2 + k_{\gamma}^2 \right) \frac{\partial D}{\partial \omega}_{\beta} \frac{\partial D}{\partial \omega}_{\gamma}}. \quad (40)$$

In this expression, A is a constant with numerical value 0.72; \hat{v}_{α} is the peak amplitude of the pump voltage; $\Gamma_{\alpha\beta\gamma}$ is given by,

$$\Gamma_{\alpha\beta\gamma} = - \frac{e\omega_{pe}^2}{2m} \left[\Pi_{\alpha} \left(\frac{1}{v_{\alpha}} \right) \sum_{\alpha} \left(\frac{1}{v_{\alpha}} \right) + \frac{\omega_{pb}^2}{\omega_{pe}^2} \Pi_{\alpha} \left(\frac{1}{v_{\alpha} - v_{0b}} \right) \sum_{\alpha} \left(\frac{1}{v_{\alpha} - v_{0b}} \right) \right]. \quad (41)$$

Computations have been carried out for Eq. (40), and are shown in Figs. 5 and 6, with $Ae\hat{V}_{\alpha}/mv_0^2$ taken as unity, i.e. for a peak pump amplitude of the order of the beam voltage. The numerical values suggest strong growth, which should be observable experimentally.

Experiment: Figure 7 shows the set-up used. Detailed descriptions of it have already been given elsewhere.^{41,68} The electron gun gives typically a beam of 20-50 V, 0.5-5.0 mA, which creates a low density plasma in the neutral background of argon at $10^{-6} - 10^{-4}$ Torr. The plasma and beam are immersed in an axial magnetic field of about 500 G, spatially homogeneous to within ± 1.5 per cent.

For launching and detecting waves in the plasma or on the beam, two probes are available together with the inner grid in the gun. One probe is fixed in axial location, and can be moved radially. The other can be moved along the plasma column. Its stem passes to the outside through a Wilson seal, and has the defect of causing pressure variations when it is moved, especially when it is moved inward. The outer grid of the gun is at ground potential, as is the plasma chamber. The inner grid is connected to a variable voltage supply and serves to regulate the extracted beam current. It is also connected capacitively to the outside, and proves to be a much more efficient detector of plasma waves than a probe. The beam voltage is determined by the setting of the gun cathode voltage.

In our preliminary experiments, connection of the inner grid to a spectrum analyzer demonstrated that the gun region was very noisy. The strength of the noise depended sensitively on the dc voltage on the inner grid. Under some conditions, parts of the gun noise spectrum were found to propagate into the plasma. Generally, it was found that a quiet gun region was associated with a beam-generated plasma with only a small axial density gradient. To avoid strong reflection of waves from the end of the plasma tube, a graphite cone was mounted on the energy analyzer.

A powerful method of determining beam and plasma parameters is to measure the wave dispersion characteristics of the system. The conventional

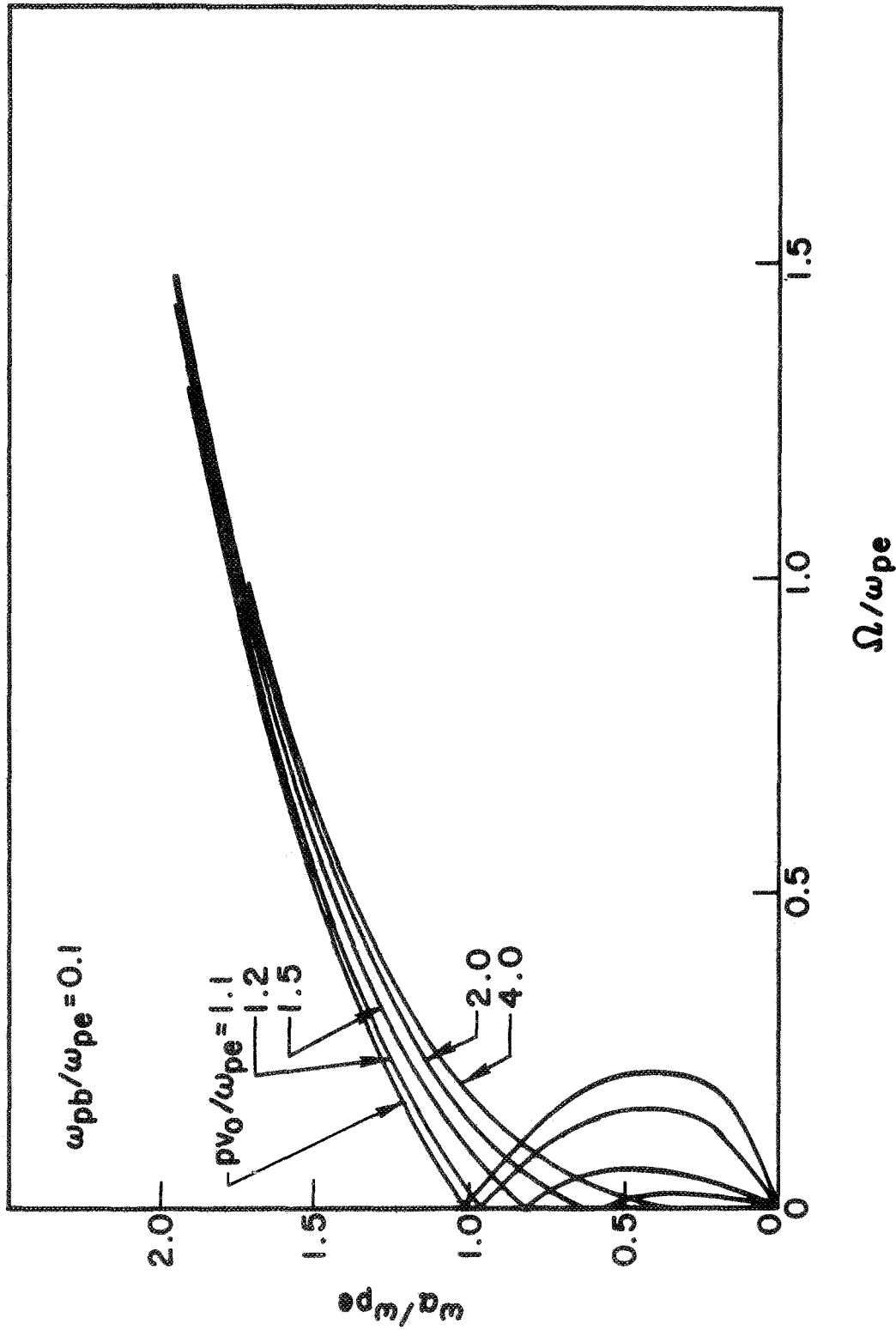


FIG. 5. VARIATION OF ω_a AND THE TEMPORAL GROWTH PARAMETER, Ω , FOR THE NONLINEARLY INTERACTING WAVES OF FIG. 4: EFFECT OF VARYING pv_0/ω_{pe} .

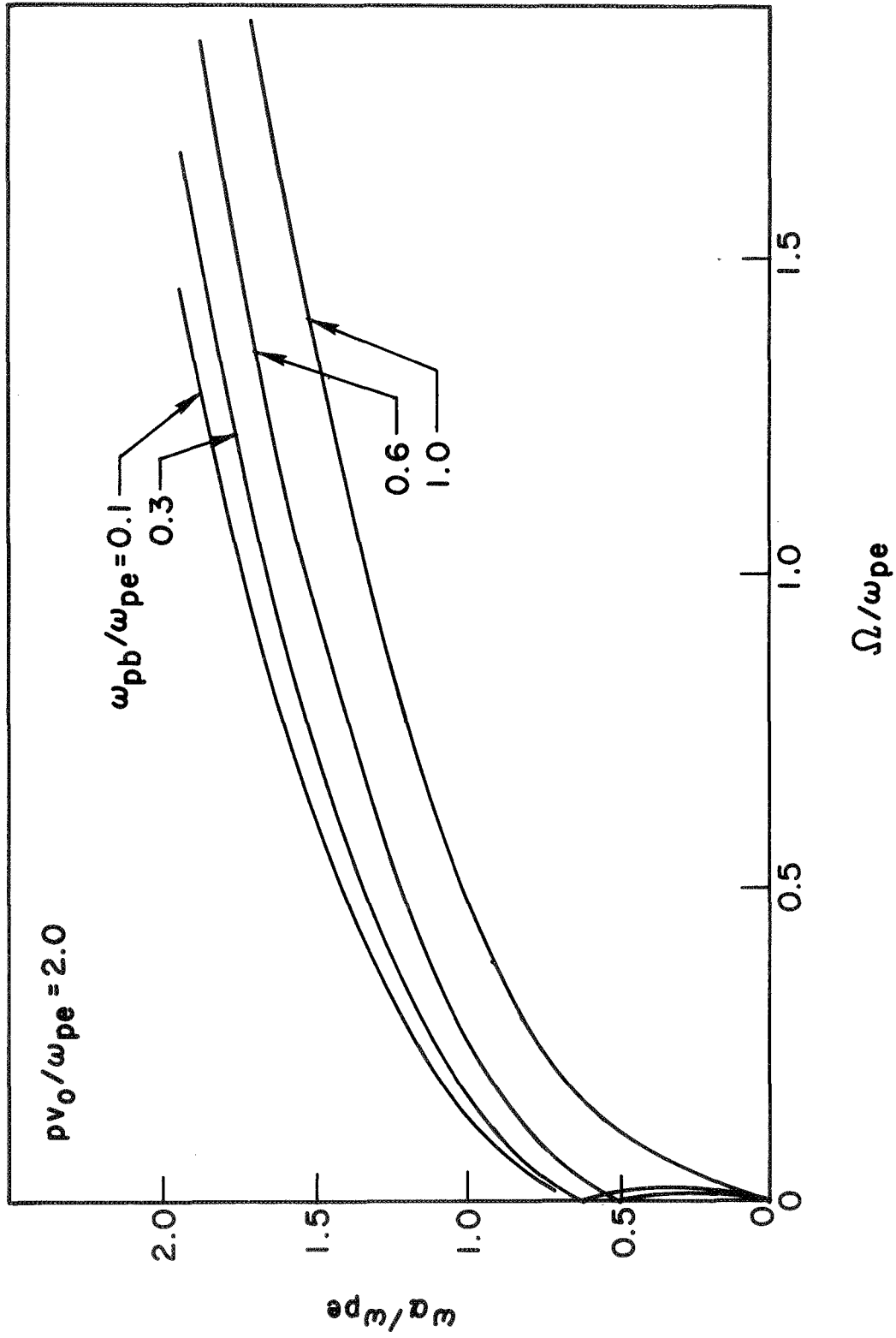


FIG. 6. VARIATION OF ω_α AND THE TEMPORAL GROWTH PARAMETER, Ω , FOR THE NONLINEARLY INTERACTING WAVES OF FIG. 4: EFFECT OF VARYING ω_{pb}/ω_{pe} .

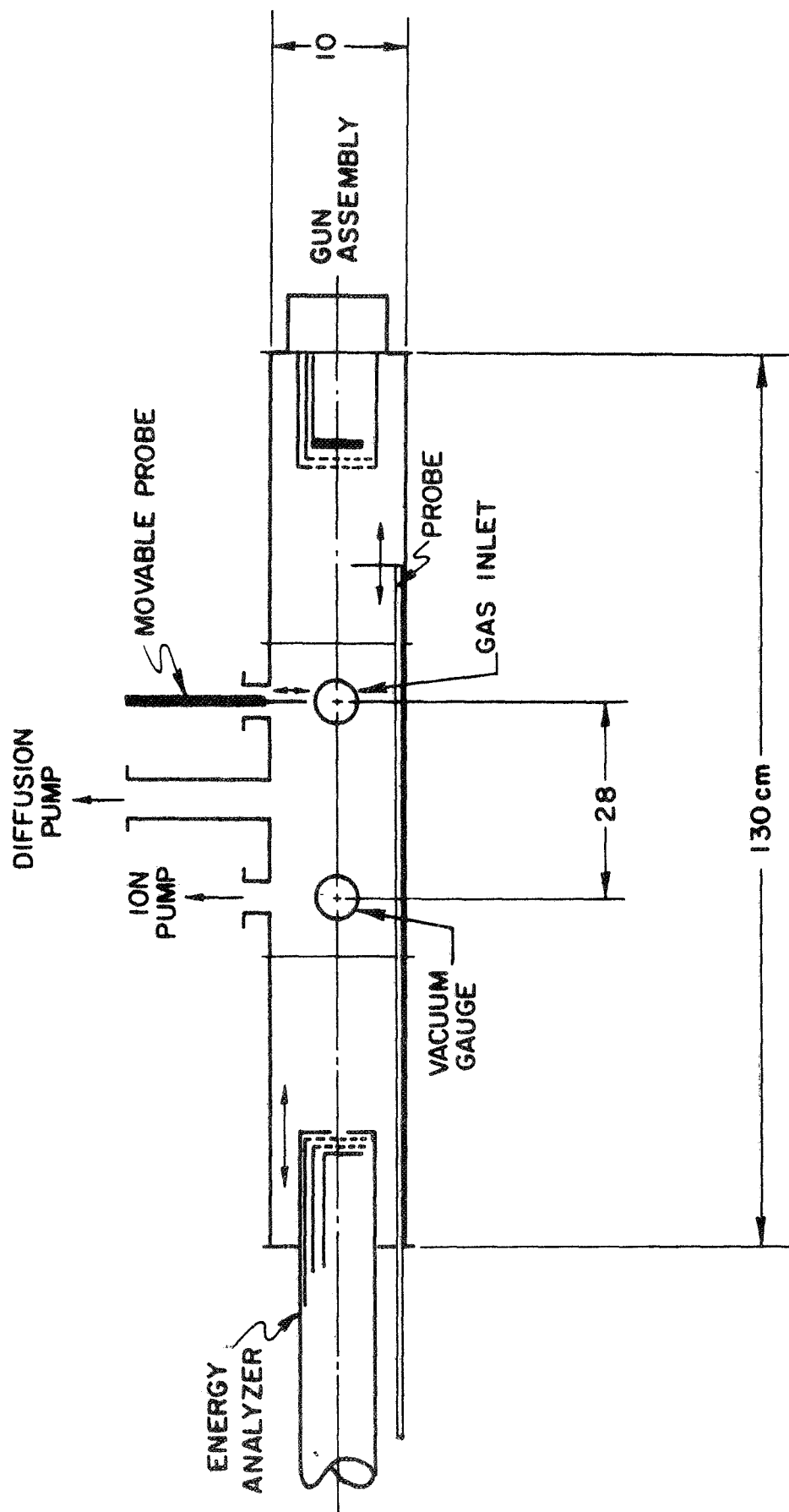


FIG. 7. EXPERIMENTAL SET-UP.

set-up used to measure the relevant phases and amplitudes is illustrated in Fig. 8. Cables A and B can be connected to either of the probes, or to the inner grid of the gun. The detector set-up as a whole is accurately square-law.

It should be emphasized that some care is needed to use the particular situation depicted in Fig. 8. Since the grid is 10 - 20 dB more effective in launching waves than is a probe, the oscillator signal must not be allowed to leak through the hybrid junction and the variable attenuator to the gun. By connecting a probe to a spectrum analyzer, it has been checked that any beam modulation due to this effect is below the detection level used in the measurements.

Another source of concern has been secondary emission from the probes, caused by beam bombardment. It was soon found that signals carried by the resulting streams of secondary electrons could be more pronounced than those propagating through the plasma. For this reason, the exciter probe is not allowed to penetrate into the beam.

Referring to Fig. 4, we see that a high frequency signal excites the two beam waves: the slow beam space-charge wave with wave number k_s , and the fast wave, k_f . These waves will interfere and produce a spatial amplitude pattern in the plasma tube, with a wave number $k_i = (k_s - k_f)/2$. For large k , $k_i \approx \omega_{pb}/v_0$, so that measurement of this quantity gives ω_{pb} directly.

To measure the plasma dispersion characteristics, it is preferable to excite with a probe Wave γ in Fig. 4, propagating towards the gun with a negative phase velocity. In the other direction, three waves are excited, one plasma wave and two beam waves. These give a double periodic interference pattern in the plasma, and correspondingly complicated measurements to interpret. Typical phase and amplitude measurements on Wave γ are illustrated by the inset in Fig. 9. In this case, the wave is launched by the axial probe, and detected by the gun grid. Some dispersion diagrams resulting from such measurements are shown in Fig. 9. The beam plasma frequency is about 2 MHz in the 0.5 mA case, and the beam dispersion curves fall closely around the dashed line. The slope of the plasma dispersion curve is $\omega_{pe}/2\pi p = 29 \times 10^6 \text{ cm}^{-1}$, and the full

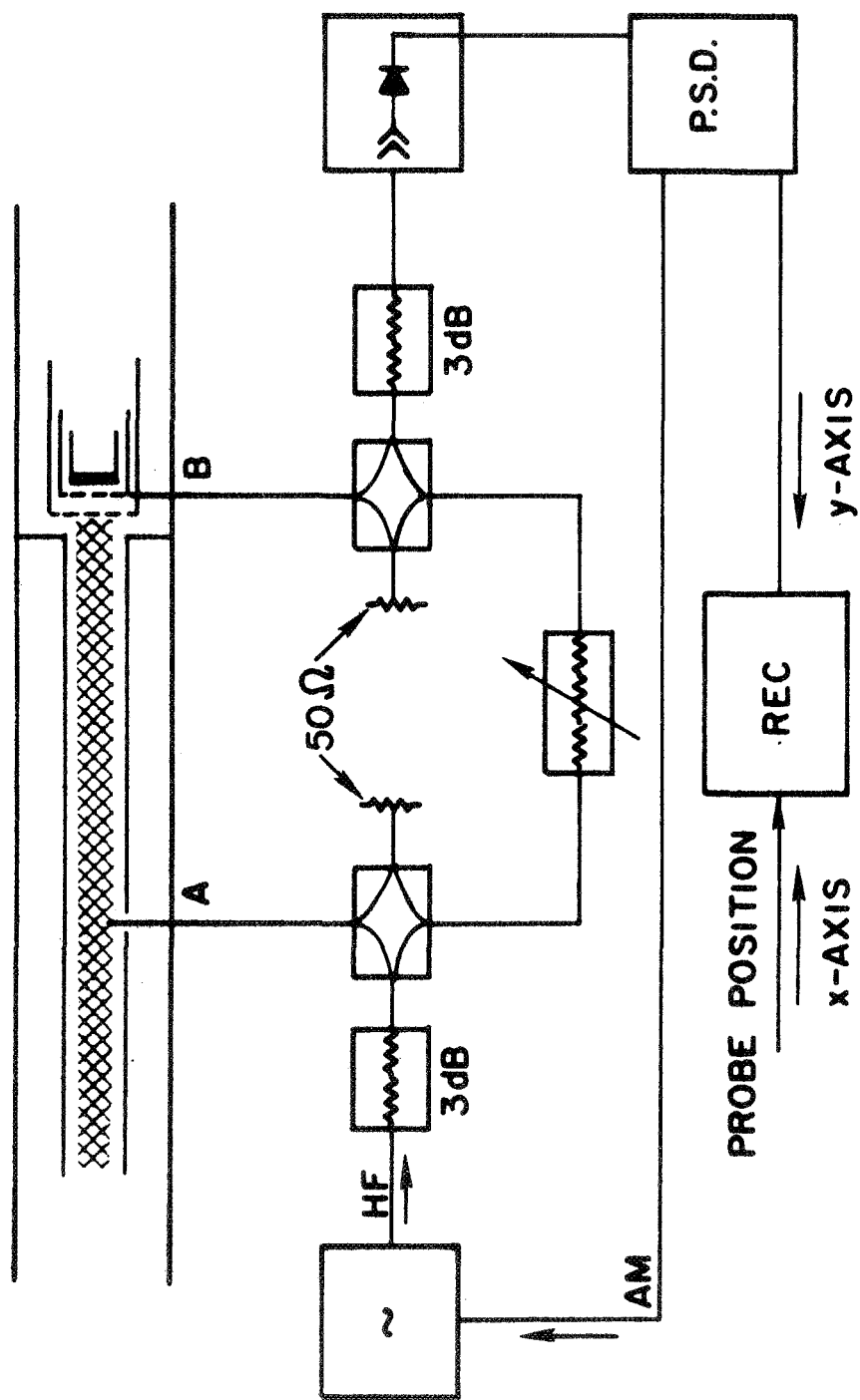


FIG. 8. MICROWAVE INTERFEROMETER.

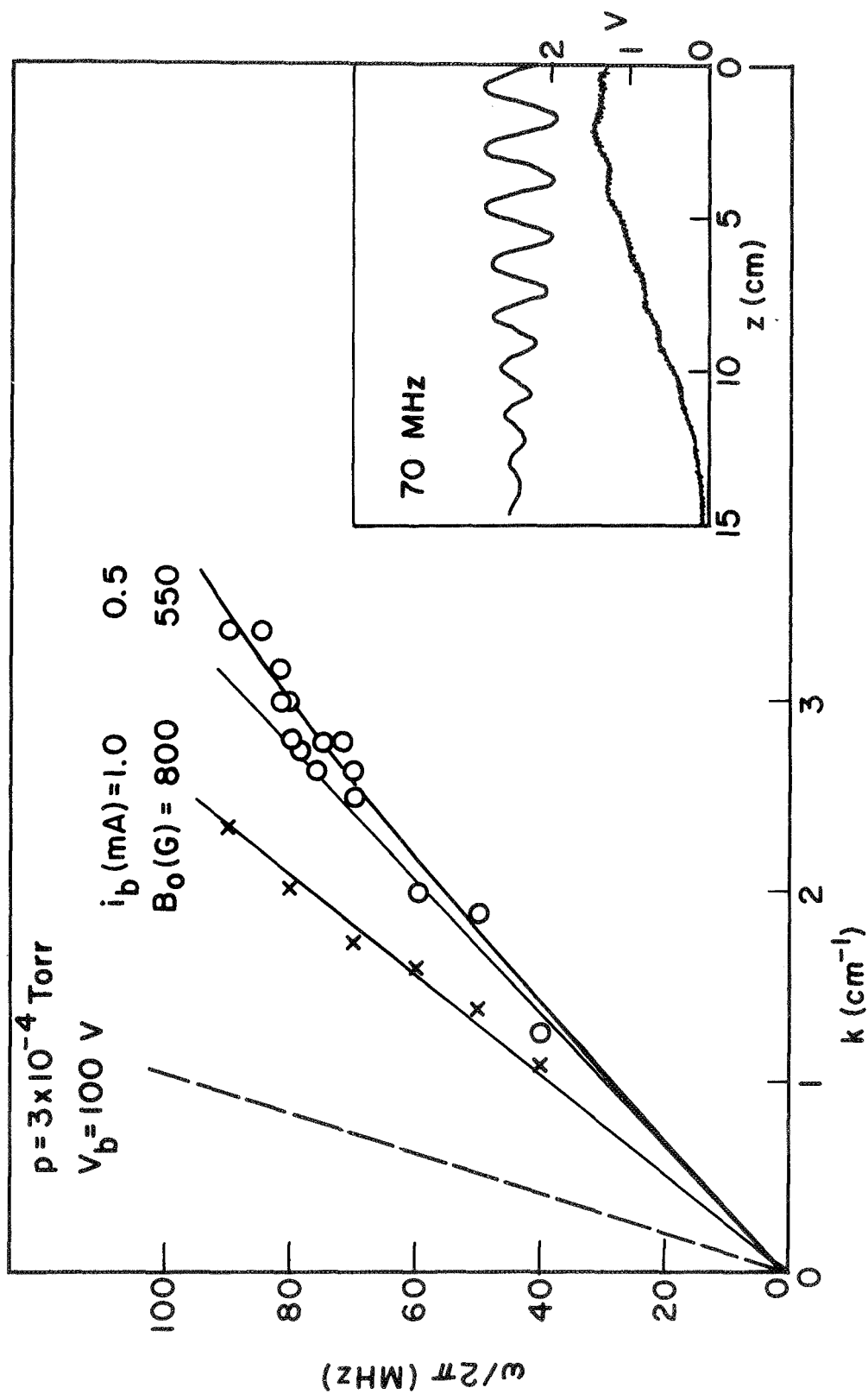


FIG. 9. DISPERSION OF PLASMA WAVE PROPAGATING TOWARDS THE GUN (WAVE γ OF FIG. 4). (THE DASHED LINE INDICATES THE DISPERSION OF THE BEAM SPACE-CHARGE WAVES. TYPICAL MEASUREMENTS OF PHASE AND AMPLITUDE ARE SHOWN INSET FOR $i_b = 1.0 \text{ mA}$, AND FIRST GRID VOLTAGE = -40 V).

theoretical dispersion curve fits through the measured points for $\omega_{pe}/2\pi \approx 200$ MHz and $p \approx 7$.

Figure 9 demonstrates that it is feasible to achieve experimental dispersion characteristics that should be linearly stable in the high frequency region. It was checked independently with a spectrum analyzer, that the plasma was indeed stable. To make the system even more stable, and to increase the range over which the experimental parameters could be varied, the plasma was surrounded by a slotted stainless steel tube 38 cm in diameter and 40 cm long, as indicated in Fig. 8. The dispersion characteristics shown in Fig. 9 were measured in this tube. Due to the gap between the tube and the gun, the plasma is inhomogeneous over the first 2 cm, and is useless for measurements. This is illustrated by the data inset in Fig. 9 which show an anomalous amplitude increase over these 2 cm, compatible with a strong density decrease. The plasma homogeneity was checked further by using the axial probe as a negatively-biased Langmuir probe, measuring the ion saturation current, i_+ . Within the narrow tube, i_+ changed by ± 2 per cent, with a probe voltage of -20 V. Similar results were obtained with a floating probe.

So far, the expected explosive instability has not been observed. There are, however, strong low frequency oscillations, and apparently parametric coupling involving these components. During the next few months, we will attempt to elucidate this problem. The project will be reported under NASA Grant NGL 05-020-176.

III. REPORTS, CONFERENCE PAPERS, AND PUBLICATIONS RESULTING FROM RESEARCH
GRANT NGR 05-020-077.

Contract Year I (1 May 1965-30 April 1966):

1. Tataronis, J.A., and Crawford, F.W., "Cyclotron and Collision Damping of Propagating Waves in a Magnetoplasma"
*Proc. 7th International Conference on Phenomena in Ionized Gases, Belgrade, Yugoslavia, August 1965 (Gradevinska Knjiga Publishing House, Belgrade 1966)
2, 244-247
*7th Annual Meeting of Plasma Physics Division of American Physical Society, San Francisco, November 1965
Bull. Am. Phys. Soc., 11, 578 (1966) [Abstract only].
IPR 27 (August 1965).
2. Crawford, F.W., "European Travel Report"
IPR 35 (October 1965).
3. Crawford, F.W., Harp, R.S., and Mantei, T.D., "RF Admittance of a Probe in a Warm Magnetoplasma"
*7th Symposium on Engineering Aspects of MHD, Princeton, N.J., March 1966.
4. Crawford, F.W., Mantei, T.D., and Tataronis, J.A., "The Plasma Capacitor in a Magnetic Field"
IPR 64 (April 1966)
Int. J. Elect. 21, 341-351 (October 1966)
5. Crawford, F.W., Harp, R.S., and Mantei, T.D., "Pulsed Transmission and Ringing Phenomena in a Warm Magnetoplasma"
*American Physical Society Meeting, Mexico City, August 1966.
Bull. Am. Phys. Soc., 11, 717 (July 1966) [Abstract only].
*IEEE International Antennas and Propagation Symposium, Palo Alto, California, December 1966
Phys. Rev. Letters 17, 626 (1966).

Semiannual Reports:

6. No. 1 (1 May - 31 October, 1965)
IPR 39 (November 1965).
7. No. 2 (1 November 1965 - 30 April 1966)
IPR 74 (May 1966)

IPR = Institute for Plasma Research Report

* = Conference Presentation

Contract Year II (1 May 1966 - 30 April 1967)

8. Crawford, F.W., Harp, R.S., and Mantei, T.D., "On the Interpretation of Ionospheric Resonances Stimulated by Alouette I"
IPR 75 (May 1966)
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Bull. Am. Phys. Soc., 11, 475 (June 1966) (Title only)
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*Fall URSI Meeting, Palo Alto, California, December 1966

Semiannual Reports:

15. No. 3 (1 May - 31 October, 1966)
IPR 111 (November 1966).
16. No. 4 (1 November 1966 - 30 April 1967)
IPR 167 (June 1967).

Contract Year III (1 May 1967 - 30 June 1968)

17. Diamant, P., "A Convenient Model of Collisions in a Plasma."
IPR 172 (June 1967)
18. Diamant, P., "Inverse Velocity Space Spectra and Kinetic Equations."
IPR 173 (June 1967)
Am. J. Phys. 35, 906-912 (October 1967)
19. Crawford, F.W., Harp, R.S., and Mantei, T.D., "Resonance Rectification Effects in Warm Magnetoplasmas"
IPR 177 (July 1967)
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[Invited Paper]. Published in, "A Survey of Phenomena in Ionized Gases" (IAEA, Vienna 1968) 109-127.
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21. Mantei, T.D., "Cyclotron Harmonic Wave Phenomena"
IPR 194 (August 1967) [Ph.D. Thesis].
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*Conference on Plasma Waves, Culham, England
September 1967 [Invited paper].
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24. Harker, K.J., Eitelbach, D.L., and Crawford, F.W., "Impedance of a Coaxial Magnetoplasma"
*American Physical Society Meeting, Pasadena, California, December 1967
Bull. Am. Phys. Soc. 12, 1137 (December 1967)
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IPR 228 (March 1968).
25. Tataronis, J.A., "Cyclotron Harmonic Wave Propagation and Instabilities"
IPR 205 (December 1967) [Ph.D. Thesis].

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*Spring URSI Meeting, Washington, D.C., April 1968
*Proc. NATO Advanced Study Institute on Plasma Waves in Space and in the Laboratory, Røros, Norway, April 1968.
Published in, Plasma Waves in Space and in the Laboratory,
Ed. J. O. Thomas and B. J. Landmark (Edinburgh Univ.
Press, Edinburgh 1970) Vol. 2, 91-109.
IPR 234 (March 1968)
27. Crawford, F. W., "A New Look at Very Long Delayed Radio Echoes"
*Spring URSI Meeting, Joint Session with AGU, Washington,
D.C., April 1968 [Invited paper].
IPR 235 (April 1968)
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*Proc. NATO Advanced Study Institute on Plasma Waves in Space and in the Laboratory, Røros, Norway, April 1968.
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Ed. J. O. Thomas and B. J. Landmark (Edinburgh Univ.
Press, Edinburgh 1969) Vol. 1, 125-156.
IPR 236 (April 1968)

Semiannual Reports:

29. No. 5 (1 May - 30 November 1967)
IPR 218 (December 1967).
30. No. 6 (1 December 1967 - 30 June 1968)
IPR 252 (July 1968).

Contract Year IV (1 July 1968 - 30 June 1969)

31. Tataronis, J.A., and Crawford, F.W., "Electrostatic Waves in Warm Magnetoplasma: I. Perpendicular Propagation"
IPR 325 (June 1969)
J. Plasma Phys. 4, 231-248 (May 1970)
32. Tataronis, J.A., and Crawford, F.W., "Electrostatic Waves in Warm Magnetoplasma: II. Oblique Propagation"
IPR 326 (June 1969)
J. Plasma Phys. 4, 249-264 (May 1970)
33. Seidl, M., "High-Frequency Beam/Plasma Interactions at Finite Temperatures"
IPR 327 (June 1969)
Phys. Fluids 13, 966-979 (April 1970)
34. Crawford, F.W., "Laboratory Plasma Resonances"
*Proc. URSI XVth General Assembly Ottawa, Canada,
August 1969 (to be published)
IPR 328 (June 1969)

Semiannual Reports:

35. No. 7 (1 July - 31 December 1968)
IPR 317 (April 1969)
36. No. 8 (1 January - 30 June 1969)
IPR 332 (July 1969)

Contract Year V (1 July 1969 - 30 June 1970)

37. Crawford, F.W., "European Travel Report"
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39. Harker, K.J., Crawford, F.W., and Eitelbach, D.L., "Impedance of a
Coaxial Magnetoplasma"
(In preparation, June 1970)
40. Hopman, H.J., "Three-wave Interaction in a Beam-plasma System"
(In preparation, June 1970)

Semiannual Reports:

41. No. 9 (1 July - 31 December 1969)
IPR 355 (February 1970)
42. No. 10 (1 January - 30 June 1970) [Final Report*]
IPR 399 (October 1970)

* Projects from this program will be reported in future under
NASA Grant NGL 05-020-176.

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